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Angles-Only, Ground-Based, Initial Orbit Determination

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14 May 1984

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FOR THE COMMANDER

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MASSACHUSETTS INSTITUTE OF TECHNOLOGY
LINCOLN LABORATORY

ANGLES-ONLY, GROUND-BASED, INITIAL ORBIT DETERMINATION

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Group 94

TECHNICAL REPORT 618

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LEXINGTON MASSACHUSETTS

Abstract

Over the past few years passive, ground-based, angles-only initial orbit determination has had a thorough analytical, numerical, experimental, and creative re-examination. This report presents the numerical culmination of this effort and contains specific recommendations for which of several techniques one should use on the different subsets of high altitude artificial satellites and minor planets.

CONTENTS

I. HISTORY AND TEST SET-UP

The long held view in astronomical circles is that Gauss solved the angles-only initial orbit determination problem when he enabled von Zach to recover Ceres from Piazzi's observations of it (made a year earlier). While Ceres was recovered by von Zach (and independently by Olbers the next night) "Gauss's" method of orbit determination had nothing to do with it (see the Introduction to Gauss 1809). About 80 years later Gibbs refined the Gaussian technique but didn't extend its range of applicability to more than nearly circular orbits. In general that is all the Gauss-Gibbs method is appropriate for. See Taff (1979b) or Moulton (1903) for a discussion of this point.

The failure of the Gauss-Gibbs technique was brought home quite forcefully during the proof of concept tests for the GEODSS (Ground-Based Electro-Optical Deep Space Surveillance) network. These were directed by the principal author (Taff 1979a). During these successful tests near-stationary artificial satellites were handled separately (Taff and Sorvari 1979a,b, 1982) and only high inclination, high eccentricity, or high mean motion ($\geq$ 1 rev/day) discoveries were dealt with by the traditional Gauss-Gibbs procedure. Since these three aspects go together for deep space satellites, that means all of the other discoveries. It failed catastrophically because we misused it. We misused it because you need to know half of the orbital element set to use it properly (in particular the semi-major axis a, the eccentricity e, and the mean anomaly at epoch M_0 or, equivalently, the time of perigee passage). But if you knew this much you are not exactly engaged in initial orbit determination, are you?

Laplace, in 1780, invented an initial orbit determination technique. It has had a very poor reputation in astronomical circles. The reasons for this are clear to us: (1) It requires a second order numerical differentiation of the observations; (2) Especially in the olden days the observations weren't very good because the observing was visual (as opposed to photographic) and neither accurate clocks nor accurate star catalogs existed yet. It wasn't until the 1930's that all of these problems were overcome but by then Gauss-Gibbs had "won" the race; and (3) In the astronomical context the method was used on slowly moving objects, certainly < 0°25/day. When all of this is coupled with a minimilast's approach (e.g., <u>three</u> sets of angles-only data), Laplace s method ought to produce very poor results.

Observational techniques and objects of interest have changed. The principal author reformulated and refined Laplace's method in the modern, high quality, data rich, fast moving object scenario (Taff 1983). Even earlier Dave Hall and he (Taff and Hall 1977, 1980) had invented a totally new initial orbit determination technique based upon "two" observations but including the angular velocities. (There's even a radar version of it in which exact initial orbit determination is reduced to the solution of a single quadratic equation.) Since one can't "observe" the angular velocity directly, the Taff-Hall method also numerically differentiates the data. But, unlike the Laplacian technique, it need do so only once and not twice. Finally, neither the resurrected Laplacian method (referred to hereinafter as the Laplace-Taff method) nor the Taff-Hall method is analytically restricted to nearly circular orbits (as the Gauss-Gibbs method is). There is no similar restriction--just questions about the accuracy (or advisability) of numerical differentiation.

Which technique should you use? Tell us what you're going to use it on first. This Report deals with the deep space population of artificial satellites ($P \gtrsim 4$ hours) observed passively from the ground, without the use of parallax techniques. Let us address this issue a bit more.

The direction of technological change--electro-optical cameras, laser radars, anti-satellite satellites, space borne surveillance systems, high value and maneuverable military satellites, coherent radars, and the decreasing time of flight of submarine launched ballistic missiles--all portend an increasing emphasis upon rapid and accurate initial orbit determination. There is no method that will suffice for all orbits, over all data sets, in all observing scenarios. Thus it becomes ever more important to search for new techniques, to delve into the physics and mathematics of old ones, and to understand all of their limitations. Note though that one can never <u>prove</u> the superiority of one technique over another by a finite set of numerical experiments. The best that one can do is to partition orbital element space, or the space of observables, into discrete portions wherein the competing (and hopefully supplementary if not complementary) methods of initial orbit determination can be ranked on the basis of performance.

How can one accomplish even this limited goal? Considering the multiplicity of potential information available a complete examination would involve a very large amount of computer time. Hence we shall separate the (laser) radar problem(s) from the angles-only case. The reason is simple--radars give distance and distance estimation is what initial orbit determination is all about. We shall also confine ourselves to the high angular speed range of the spectrum thereby eliminating natural bodies from consideration.

Thus, and not totally accidentally, the objects of interest are deep space or high altitude artificial satellites. Furthermore, we restrict this discussion to the non-parallax type of data acquisition.

In such circumstances, for such objects, three initial orbit determination procedures merit consideration. One is the Gauss-Gibbs method. It is included because of its reputation, not because we are promoting it. The principal method we now use on the faster moving subset of the deep space artificial satellite population is Taff's modification of Laplace's method. The third technique considered here is the Taff-Hall angular velocity method. Now, the latter two schemes presume the observational capability to rapidly acquire large amounts of high quality data. This information will then be smoothed in some fashion and ultimately differentiated. Because of this, and the fact that the Gauss-Gibbs method is per force restricted to three sets of angles-only data, a way has to be found to try to balance the scales. We have done this as follows: We have selected a subset of orbital element sets from the deep space population (as it existed on a certain date in mid-1982). We have used this subset of the actual population's element sets to generate topocentric position vectors for a particular geographical location (the CONUS observatory of the GEODSS network but that's not relevant). The time spacing between position vectors is two minutes--the baseline specification for the GEODSS program. From this set of passes from the subset of elements we chose, we then selected 96 passes of 25 different satellites. For the midpoint of each pass we computed the radius of convergence and of the f and g series. (See Taff 1979b.) In each case this figure was then reduced by 20% and eight-tenths of the radius of convergence, rounded down to the nearest whole two

minutes, defined the total time span of the data. For the tests of the Gauss-Gibbs method the input data are topocentric right ascension, declination, and time at the beginning, middle, and end of the time interval just defined. For the tests of the other two methods the input data consisted of all of the (A,Δ,t) triplets within the time interval unless there were more than $\sim$ 12-15 of them. In the case of the larger radii of convergence only every other (or every third, etc. [if need be]) position vector was regarded as known. In this fashion, while denying the Laplace-Taff and Taff-Hall techniques data that would be available in practice, we've tried not to tip the scales too much in their favor.

The above mentioned passes and orbital element sets are weighted towards the stressing cases, i.e., the high eccentricity, fast moving portion of the hundreds of passes originally generated. This, however, merely reflects the actual state of affairs. Also the number of data points reduction scheme outlined above kept the total number of assumed data points to be at most 15. All of the data had the same number of significant decimal digits--namely right ascensions to the nearest tenth of a second of time and declinations to the nearest whole second of arc. In order to complete the analysis the entire test should be redone at at least one, and preferably at two other levels of precision. This would provide sorely needed information about the robustness of the three procedures. A part of this has been completed and is discussed below.

Now that we've outlined the ground rules what criteria should be used to measure the efficacy of the three competing techniques? We could define some six dimensional norm in orbital element set space and compare each

initial orbital element set to the real one (e.g., the one used to generate the pseudo-observational data). If we knew what metric to use to define a meaningful norm, and if we knew how large "bad" was or how small "good" was, then we would consider this alternative. Not being in possession of such knowledge we prefer to test the generated orbital element sets in their natural mode of utilization (in the angles-only context--a nice touch of self-consistency); pointing a telescope. Therefore, the measure that we have used is angular error, on the topocentric celestial sphere, between the position from the computed orbital element set and the position from the real orbital element set. More than this, such pointing predictions ("look angles" if you must) have been generated at intervals of 0.5, 1, and 2 hours into the future (sometimes in the past; Newton's equations of motion are time reversible). Therefore, for each pass of each satellite we could have nine angular errors as measured on the topocentric sky--an error from each of three initial orbit determination schemes at three different prediction intervals (not all passes of all satellites were run each time). This information has been reproduced in Table I. The units are minutes of arc.

II. SOFTWARE AND PROCEDURES

The Gauss-Gibbs software used is the standard GEODSS ETS code (Taylor 1978). We have no doubts about this. The Laplace-Taff and Taff-Hall software were designed, debugged, and tested by us for this Report. There exists a heliocentric version of the Laplace-Taff code which is used in our Earth-approaching asteroid search work (Taff 1981). In the Gauss-Gibbs case the flexibility within the interactive part of the program was circumvented because only three observations were entered. In the Laplace-Taff case there are two other degrees of freedom. Since the topocentric right ascensions and declinations are independently least squares fit to quadratic, . . ., quintic polynomials one can choose (say) the cubic right ascension fit as the best one for that variable and (say) the quartic declination fit as the best one for that variable. "Best" is difficult, if not impossible to define since the addition of another free parameter necessarily reduces the sum of the squares of the residuals. We adoped a rule of one-third; if increasing the degree of the polynomial to which the data were fit did not decrease the sum of the squares of the residuals by at least a factor of three, then the previous, lower degree fit was "best". We never tested polynomials of degree six or higher.

In the Taff-Hall case the problem is more complicated. The results of the aforementioned least squares fits for A (it's identical for the declination) are coeval values for A, $\dot{A}$, and $\ddot{A}$. This is the "raw" data one needs for Laplace's method. Their simultaneous epoch is the average observation time. For the Taff-Hall case one needs two sets of A, Δ, $\dot{A}$, and $\dot{\Delta}$ with the epochs of all the variables in each separate set the same (say t_1 and t_2).

Now, the behavior of the expectation of the variance of the rates depends upon the degrees of the polynomial fits (see Taff 1983 and the Appendix). Optimally, since t_1 and t_2 are at our disposal, one would use those values for which the expected variances of A, Δ, $\dot{A}$, and $\dot{\Delta}$ were minima (if such existed--in general one doesn't; again see the lengthy discussion in Taff 1983). These optimal times were computed for the cubic, quartic, and quintic fits for both the positions and their rates in the simplified case of equally weighted data, symmetrically observed about the midpoint, and as the number of observations approaches infinity. We did keep the leading terms in the latter quantity for low order fits. These results are in the Appendix to this Report and show that one can retain the element of freedom of choosing different polynomial fits for the declination and right ascension, without excessive loss of precision, if one does it carefully. Clearly without this analysis one would be shooting in the dark. The bottom line here is that if the data extend (uniformly) from $t = -T$ to $t = +T$ then t_1 can be at $-T/6$ and t_2 can be at $+T/6$. This choice is a combination of this rather involved expected variance analysis and simplicity, but mostly the former. (The rigorous results for the cubic fit are $\pm T/\sqrt{7} = \pm 0.38T$, for the quartic fit $\pm T/\sqrt{3} = \pm 0.58T$, and for the quintic fit $\pm 0.25T$. The expected variances for the positions are relatively flat out to $\sim \pm T/2$. See the figures in the Appendix). Hence, utilizing the same residual "significance" scheme mentioned above, the Taff-Hall case was treated in a fashion very similar to the Laplace-Taff case.

So far we've explained the data reduction. Next we need to treat the construction of the orbital element set. For the Gauss-Gibbs method this is a classic problem. If the solution is not in Taylor (1978) then see any celestial mechanics book. For the Laplace-Taff case one winds up with $A, \Delta, \dot{A}, \dot{\Delta}, \ddot{A}, \ddot{\Delta}$ as input and $R, \dot{R}$ (topocentric distance and radial velocity) as output. Knowing the observer's geocentric location and velocity one uses R, A, Δ and $\dot{R}, \dot{A}$, and $\dot{\Delta}$ to obtain a geocentric location and velocity (at $t = 0$ in the above scenario). Now it's a straightforward algebra problem to go from $\underline{r}$ and $\dot{\underline{r}}$ to an orbital element set. See, for instance, Brouwer and Clemence (1961).

Once again the Taff-Hall method is different. The output are two sets of R and $\dot{R}$ and hence two orbital element sets can be computed. We only used one: t_1 if we were predicting the past for the satellite, t_2 if we were predicting the future. (Some of our 96 passes are for satellites in the act of setting relative to the observatory and our pointing program, which generated the pseudo-observational data, stopped producing at the instant of setting. In these geometrically interesting passes we predicted the past.)

Finally we did not run every pass of every satellite through each initial orbit determination procedure. It would be a total waste of time to have done so for the near-stationary artificial satellites because they don't move rapidly enough to meaningfully smooth observations thereof and neither of these three techniques is the optimal one for this class of artificial satellite. The optimal technique for near-stationary artificial satellites is NSDC (for Near-Stationary Differential Corrector) alluded to above (Taff and Sorvari 1979a, 1982). Keeping to the high angular speed range of the passes we

selected, the Laplace-Taff procedure was executed on 29 passes of 14 different satellites. The Taff-Hall method was run on a larger set -- 30 passes of 11 different satellites.

A. Low Precision Data Tests

All of the above discussion, and most of Table I below, refers to high quality data. This means positions good to 1". This is much better than the GEODSS network can provide. We added enough random noise to the "data" to increase the precision to 5" and redid 26 passes on 14 different satellites for both the Gauss-Gibbs technique and the Laplace-Taff technique (no low precision Taff-Hall runs were performed). In general the Gauss-Gibbs technique degraded only slightly in performance while the Laplace-Taff did so by factors of two to three (or larger) relative to its high precision data performance. This should not surprise you and would be partly compensated for by having a higher data rate. Presumably the Taff-Hall technique would be worse too, but not fall off as rapidly as the Laplace-Taff method did as it only involves a single numerical differentiation of the data.

Let us forcefully reiterate that the correct conclusion from this subset of tests is not that the Gauss-Gibbs method is the algorithm of choice. This Report uniformly rejects the Gauss-Gibbs technique except as the method of last resort.

TABLE I

PREDICTION RESULTS

Satellite Information		Time Spans	High Precision		Low Precision		Angular Speed ("/sec)
			Gauss-Gibbs	Laplace-Taff	Gauss-Gibbs	Laplace-Taff	
SDC #	11926						
n	1.02	$0^{h}.5$	$0'.14$	$0'.03$	∞	$0'.2$	
e	0.331	1.0	0.46	0.04	∞	0.7	32.10
i	$10^{\circ}.44$	2.0	0.58	0.13	∞	2.0	
SDC #	12137						
n	2.37	0.5	5.25	0.76	18.7	15.5	
e	0.724	1.0	4.50	1.20	4.0	1.6	66.66
i	$46^{\circ}.73$	2.0	4.20	2.05	3.0	2.8	
SDC #	12679						
n	3.49	0.5	4.94	0.07	5.7	4.0	
e	0.626	1.0	4.15	0.03	3.3	3.9	32.00
i	$89^{\circ}.94$	2.0	2.34	0.21	8.2	8.4	
SDC #	12679						
n	3.49	0.5	6.02	0.78	6.4	2.3	
e	0.626	1.0	5.70	3.85	7.4	11.4	68.20
i	$89^{\circ}.94$	1.5	5.06	8.72	8.6	26.1	
SDC #	83781						
n	2.01	0.5	9.41	∞			
e	0.735	1.0	9.94	∞			112.80
i	$64^{\circ}.55$	2.0	13.47	∞			
SDC #	83871						
n	2.01	0.0	10.69	0.07	∞	1.1	
e	0.735	1.0	8.74	8.33	∞	4.9	136.73
i	$64^{\circ}.55$	2.0	9.79	21.57	∞	14.2	

TABLE I (Continued)

Satellite Information		Time Spans	High Precision Gauss-Gibbs	High Precision Laplace-Taff	Low Precision Gauss-Gibbs	Low Precision Laplace-Taff	Angular Speed ("/sec)
SDC #	83878						
n	1.99	$0^{h}.5$	3".68	3".34	3".5	6".8	
e	0.740	1.0	4.72	6.30	4.4	11.0	113.89
i	62°.87	2.0	7.44	9.21	6.7	17.2	
SDC #	83878						
n	1.99	0.5	6.75	6.75	7.1	15.5	
e	0.740	1.0	2.02	1.34	2.3	2.2	54.42
i	62°.87	2.0	2.85	1.77	3.5	4.5	
SDC #	83878						
n	1.99	0.5	5.87	15.58	5.7	16.8	
e	0.740	1.0	2.74	4.63	2.7	5.0	35.50
i	62°.87	1.5	4.59	8.98	4.5	9.7	
SDC #	83878						
n	1.99	0.5	2.59	0.42	2.5	0.6	
e	0.740	1.0	1.36	0.51	1.0	0.8	164.60
i	62°.87	2.0	0.66	1.39	1.6	0.8	
SDC #	83878						
n	1.99	0.25	3.22	0.52	8.0	9.3	
e	0.740	1.0	4.18	0.63	1.7	3.0	68.97
i	62°.87	2.0	18.71	1.96	1.3	10.4	
SDC #	10167						
n	1.99	0.5	1.98	0.58	2.0	2.8	
e	0.652	1.0	2.96	1.86	3.7	5.7	73.74
i	64°.28	1.5	3.88	2.83	4.1	8.9	

TABLE I (continued)

Satellite Information		Time Spans	High Precision Gauss-Gibbs	High Precision Laplace-Taff	Low Precision Gauss-Gibbs	Low Precision Laplace-Taff	Angular Speed ("/sec)
SDC #	10167						
n	1.99	$0^h.5$	2!17	0!62	2!4	0!5	
e	0.652	1.0	1.28	0.57	0.2	0.9	42.10
i	64°28	2.0	3.44	3.22	2.8	4.2	
SDC #	12996						
n	2.04	0.5	2.68	3.74	2.7	3.1	
e	0.685	1.0	1.86	4.49	1.9	8.8	59.40
i	61°35	2.0	0.44	4.88	0.8	31.0	
SDC #	83746						
n	2.01	0.5	5.77	NO			
e	0.660	1.0	7.80	NO			132.92
i	71°11	2.0	18.27	NO			
SDC #	83601						
n	2.13	0.5	9.20	17.29	NO	23.9	
e	0.710	1.0	10.34	47.50	NO	74.0	130.20
i	64°16	2.0	13.92	∞	NO	∞	
SDC #	83744						
n	2.46	0.5	3.20	0.66	1.4	3.4	
e	0.715	1.0	7.83	1.88	6.2	17.2	8.10
i	10°32	2.0	∞	6.83	∞	∞	
SDC #	83744						
n	2.46	0.5	0.71	7.97	0.9	8.4	
e	0.715	1.0	1.05	11.69	1.5	17.3	158.51
i	10°32	2.0	1.83	16.65	13.3	38.6	

TABLE I (continued)

Satellite Information		Time Spans	High Precision		Low Precision		Angular Speed ("/sec)
			Gauss-Gibbs	Laplace-Taff	Gauss-Gibbs	Laplace-Taff	
SDC #	83885						
n	2.27	$0^{h}.5$	$3''.86$	$0''.80$	$2''.5$	$2''.2$	
e	0.732	1.0	11.19	3.40	6.8	10.1	22.91
i	$27^{\circ}.40$	2.0	39.30	13.83	26.4	43.6	
SDC #	83885						
n	2.27	0.5	2.03	0.70	2.2	0.4	
e	0.732	1.0	25.37	5.84	1.5	8.2	10.08
i	$27^{\circ}.40$	2.0	∞	63.72	27.6	∞	
SDC #	898						
n	2.01	0.5	5.94	NO			
e	0.643	1.0	6.22	∞			174.00
i	$71^{\circ}.28$	2.0	7.73	∞			
SDC #	898						
n	2.01	0.5	1.63	0.35	2.0	0.3	
e	0.643	1.0	3.12	2.11	3.5	0.9	8.50
i	$71^{\circ}.28$	2.0	13.66	15.61	12.3	4.7	
SDC #	898						
n	2.01	0.5	4.54	0.11	4.4	1.2	
e	0.643	1.0	5.44	0.42	3.6	∞	19.83
i	$71^{\circ}.28$	2.0	14.31	1.56	6.3	∞	
SDC #	83750						
n	4.22	0.5	2.84	0.65	2.8	1.8	
e	0.593	1.0	3.91	1.81	4.0	4.8	34.42
i	$27^{\circ}.14$	2.0	10.63	4.19	11.3	11.2	

TABLE I (continued)

Satellite Information		Time Spans	High Precision Gauss-Gibbs	High Precision Laplace-Taff	Low Precision Gauss-Gibbs	Low Precision Laplace-Taff	Angular Speed ("/sec)
SDC #	83750						
n	4.22	0.5	3.74	2.08	5.0	5.1	
e	0.593	1.0	4.49	6.61	6.8	30.3	22.43
i	27°14	2.0	78.31	42.26	87.5	∞	
SDC #	83750						
n	4.22	0.5	1.38	2.98	5.5	5.8	
e	0.593	1.0	10.99	11.24	14.6	12.4	59.63
i	27°14	2.0	29.94	52.46	53.6	44.4	
SDC #	83887						
n	2.29	0.5	1.36	1.23	3.5	4.3	
e	0.723	1.0	1.66	3.65	6.3	13.5	66.80
i	47°39	2.0	3.89	9.88	14.6	37.3	
SDC #	83887						
n	2.29	0.5	0.50	0.69	NO	NO	
e	0.723	1.0	4.02	2.67	NO	NO	146.29
i	47°39	2.0	13.05	7.89	NO	NO	
SDC #	83887						
n	2.29	0.5	0.38	0.25	NO	1.6	
e	0.723	1.0	3.80	1.25	NO	6.5	8.91
i	47°39	2.0	16.90	9.28	NO	51.8	

III. RESULTS

The Table contains the detailed results of the 1/2, 1, and 2 hour predictions for the 29 passes of the 14 different artificial satellites seen through the Laplace-Taff algorithm. Results are given, for each pass, for the Gauss-Gibbs method and the Laplace-Taff method. On the right hand side of the Table the low precision data results are presented. The listed topocentric units are minutes of arc. No entry means no computation. An entry of NO means a successful run but no physical orbital element set was produced. Errors in excess of 100' are considered to be infinite and are listed as ∞. We believe that the worst one was 9494'.

A. Gauss-Gibbs

This test is not designed to highlight Gaussian orbit determination. Rather, it was designed to see if the Gauss-Gibbs method could ever work well on fast moving artificial satellites in eccentric orbits. (It should work on objects in nearly circular orbits.) However, for the first time proper cognizance of the essential restrictions that are part and parcel of Gaussian initial orbit determination have been included. We shall let the numbers speak for themselves. We do not recommend the use of the Gauss-Gibbs angles-only method. For the 29 passes detailed in the Table the average positional errors at 0.5, 1, and 2 hours (and the standard deviation about their means) were $3'.95 \pm 2'.70$, $5'.58 \pm 4'.91$, and $12'.56 \pm 16'.01$. For the other passes in the original sample the same quantities are $1'.83 \pm 2'.34$, $2'.23 \pm 4'.03$, and $6'.25 \pm 16'.05$. For the low precision data the 29 pass Gauss-Gibbs set results are $4'.52 \pm 3'.80$, $4'.16 \pm 3'.17$, and $14'.40 \pm 21'.33$. Invisible in these statistics

is the failure rate of the Gauss-Gibbs technique even though we've observed all of the analytically necessary restrictions. This rate was negligible for the high precision data runs (both groups) but approached 1/3 for the low precision runs.

B. Laplace-Taff

We have computed, for the half hour, the hour, and the two hour prediction intervals, the ratio of the positional error of the Gauss-Gibbs method to that obtained from the Laplace-Taff method. The larger this number is, the worse Gaussian initial orbit determination fares relative to Laplacian initial orbit determination. The three averages, and their standard deviations about their means, are 8.0 ± 27, 13.2 ± 32, and 2.4 ± 3.2. The accuracy of the Laplacian data alone, in the same format as given for the Gauss-Gibbs technique is $2'.65 \pm 4'.53$, $5'.15 \pm 9'.22$, and $12'.44 \pm 16'.48$. For the low precision data Laplace-Taff runs the appropriate numbers are $5'.48 \pm 6'.23$, $7'.85 \pm 6'.92$, and $18'.98 \pm 16'.93$. The Laplace-Taff method failure rate on the poorer data is about half that of the Gaussian.

From our point of view the ratio numbers tell the real story. Laplace-Taff is an order of magnitude better than is Gauss-Gibbs without having to know half of the orbital element set before starting the computation. Higher data rates will improve its performance.

C. Taff-Hall

The Taff-Hall technique rests on the fact that the topocentric expression of angular momentum conservation and energy conservation is a quartet of equations involving the position $\underline{\mathcal{L}}(A,\Delta)$, the angular velocity $\dot{\underline{\mathcal{L}}}(A,\Delta,\dot{A},\dot{\Delta})$, the topocentric distance R, and the topocentric radial velocity $\dot{R}$.

The key is to use the fact that these are constants of the motion and write

$$\underline{L}_1 = \underline{L}_2, \quad E_1 = E_2 \tag{1}$$

at two times t_1, t_2. There are now 12 unknowns in these four equations. In the radar case eight quantities (A,Δ,R, and $\dot{R}$ at both times) are measured so that the problem is well posed. In the high angular speed case eight quantities are known too ($A,\Delta,\dot{A}$, and $\dot{\Delta}$ at both times).

The early (Taff and Hall 1977) solution method was a 4 dimensional Newton-Raphson technique. This procedure was not robust, didn't exploit the analytical simplicity of Eqs. (1), and didn't work well. When we started this work we switched to a steepest descent method which was not robust, did exploit some of the analytical simplicity of Eqs. (1), and didn't work well. Finally we have fully exploited the analytical simplicity of Eqs. (1) to reduce the problem to a 1 dimensional one. In particular we have reformulated the system so as to (appear) to be a single equation (for E_1-E_2) in one unknown (R_2). From an assumed R_2 we compute R_1 (by explicit algebra) and then $\dot{R}_1$ and $\dot{R}_2$ (similarly). This requires the use of three equations and the angular momentum conservation equations were utilized. Next we use all of this to compute E_1-E_2 (which should vanish). If $E_1-E_2 \neq 0$ then we increment R_2 and repeat until E_1-E_2 changes sign. We then home in on the root by decreasing the R_2 step size by a factor of ten and changing direction. We use $\Delta R_2 = 0.01$ Earth radii, start at $R_2 = 1$ and go out to $R_2 = 10$ (we know we're doing high altitude satellite initial orbit determination) before declaring a failure.

Of the 30 passes of 15 different artificial satellites we tried no root was found in 14 cases, in 10 cases a root was found but no physical

orbital element set was generated, and in 6 cases both a root was found and a physically sensible orbital element set was produced. The reason for the first group appears to be the extraordinarily large slope in the E_1-E_2 vs. R_2 relationship. An explanation for the middle group is the existence of multiple roots and our a priori inability to guess the correct one (for if we knew R_2 then . . .). The last group represents the "successes" but the orbital element sets are so poor that we've not formally computed the pointing errors. Without a clearer and deeper understanding of the sensitivities of this method we must regard it as a failure. We suspect that it will do much better with a higher data rate than one observation per two minutes or that it is appropriate for use on moderate (as opposed to high) angular speed artificial satellites. As supporting evidence, the average angular speeds for the three Taff-Hall subgroups are 94"/sec, 85"/sec, and 40"/sec.

IV. RECOMMENDATIONS

A. Artificial Satellites

While we have tested the three different methods of initial orbit determination in an objective, unbiased fashion, we have used a priori information concerning the orbital element sets in constructing the tests. This has biased the results towards the Gauss-Gibbs technique. Even so, when compared to the Laplace-Taff algorithm (for this sample of this subset of the deep space artificial satellite population with a one per two minute data rate of arc second precision data), it fares poorly--by a factor of ~ 10. The Taff-Hall technique must be regarded as a failure too.* However we do not recommend the Laplace-Taff method to the exclusion of all others exactly because--for the existing deep space population--one can tell almost certainly what type of satellite one is observing.

The deep space catalog on the day we began this work had 540 entries in it. Of these 16% were Cape Canaveral rocket bodies ($n \approx 2$ rev/day, $i \approx 29°$, $e \approx 0.7$), 31% were near-stationary satellites ($0.9 \lesssim n \lesssim 1.1$ rev/day, $e \lesssim 0.15$, $i \lesssim 15°$), and 40% were Molniya-type ($n \approx 2$ rev/day, $i \approx 63°$, $e \approx 0.7$, $\omega \approx 285°$). These three types are easily discernable <u>at the telescope</u> and comprise 87% of the catalog. Therefore if one is willing to take the (small) risk of making a mistake, one can pick and choose the technique (amongst Laplace-Taff, NSDC, or Gauss-Gibbs) in an appropriate fashion. This is our first recommendation.

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*We suspect that it can be made much more robust and demonstrated successfully on moderate (20-50"/sec) angular speed artificial satellites. Since this endeavor has dragged on for two years we thought that publishing this now was the prudent thing to do.

A step back from these strong assumptions is the less restrictive division of deep space artificial satellites into 3 angular speed ranges.

I. topocentric angular speed $\gtrsim$50"/sec:
   use Laplace-Taff;

II. topocentric angular speed ε ($\sim$20"/sec, 50"/sec):
   if near equator use Laplace-Taff, if nearer the poles (say above or below 20°) use Gauss-Gibbs;

IIIA. topocentric angular speed $\lesssim$ 20"/sec and not near-stationary: use Gauss-Gibbs;

IIIB. topocentric angular speed $\lesssim$ 20"/sec and near-stationary: use NSDC.

This is our second level recommendation. We hope to be able to replace the recommendations for the moderately moving objects with an improved Taff-Hall algorithm.

B. Asteroids

From our perspective there are three types of asteroids. The huge majority (99%) are main-belt minor planets. They are in nearly circular orbits of slight inclination with a period $\sim$ 4.5 yr. Their excursions through orbital element space (the argument of perihelion values are uniformly distributed on [0,360°] while their longitudes of the ascending nodes cluster near Jupiter's) and through the solar system are minimal. The ones we have actively searched for (Taff 1981) are in high eccentricity, high inclination orbits and traverse large parts of the inner solar system. About 60 of these have been found in the last two decades. The last group is the $\sim$ 50 other high eccentricity or high inclination objects that have longer periods. (The Trojans are a special

case and Chiron is unique). As initial orbit determination is interesting only for the latter two groups, and their eccentricities are neither so high nor their periods so short as to preclude a Gaussian type technique, one may as well use the Gauss-Gibbs method. Of far more importance is the independent acquisition of precise data.

C. Characteristics of the Orbital Element Sets

The uses to which one might put an orbital set are myriad. Foremost amongst them are pointing a telescope, pointing a radar, preventing surveillance, or planning a rendezvous. In all these and many other instances, the accuracy of the individual components of the element sets is not at issue. The techniques discussed above have certain characteristic systematic correlations amongst the osculating elements generated by their use. Typically they all determine the orbital plane (i.e., the direction of $\underline{L}$) very well and this aspect will not be discussed further.

When Laplace's method is utilized in a data-rich scenario, so that smoothing and analytical differentiation of the interpolating polynomial can be performed, the orbital element sets that it produces are unbiased. They are frequently amazingly good, each element separately within 0.01% of its true value. The method is not robust to decreasing the precision of the observational data but this can be somewhat compensated for by decreasing the time intervals between the observations and increasing the number of them. This is the best of the techniques we've used.

Gauss's method typically involves a mean motion/eccentricity swap that reproduces the interpolating arc well but is not capable of accurate extrapolation. Corollary effects are usually seen in the argument of periastron

and in the time of periastron passage. The method is relatively robust to the degradation of the data. Of course, all of these statements presume that the radius of convergence stricture is being observed a priori.

When a physically meaningful ($a > 0$, $e \in [0,1]$) orbital element set is computed by the Taff-Hall technique, it typically is a biased one, although not as severely as one produced by Gauss's method (for the same data). The method is not robust and frequently does not yield a physically meaningful element set at all. Neither this nor Gauss's method can be recommended for use.

The most extensively used partial knowledge method is the one developed in Taff and Sorvari (1982) for near-stationary artificial satellites. Within its constraints it works without bias and is extremely robust. We conjecture that the more assumed a priori about the orbital element set, and the more accurate these assumptions, the better the performance of the technique. The Minor Planet Center uses a comparable approach for main-belt asteroid initial orbit determination.

APPENDIX*

In Taff (1983) a detailed analysis of constant, linear, quadratic, and cubic least squares fitting was provided. In particular, the estimated variances of the first and second derivatives of the observed quantity were evaluated. As the order of the fit becomes higher, the algebra necessary to complete the analysis becomes increasingly involved and tedious to carry through. Three simplifying assumptions greatly speed this process. They are

1. That the data are acquired symmetrically (in time) about some central epoch t_0. Then the use of $\tau = t - t_0$ is preferred. The observations of x at $t = t_n$ $(\tau = \tau_n)$ are labeled as x_n. The data are also presumed to have equal weight w.

2. The more restrictive assumption that the time spacing between the observations are all equal (to T). Then

$$\{\tau_n\} = \tau_{-N}, \tau_{-N+1}, \ldots, \tau_0 = 0, \tau_1, \ldots, \tau_N$$

for the total of 2N + 1 data points are just given by

$$\tau_n = (n - N - 1)T \qquad n = -N, -N+1, \ldots, 0, \ldots, N$$

3. That the asymptotic limit as $N \to \infty$ is taken and only the leading terms are kept in all pertinent expressions.

The necessity of two epochs for the Taff-Hall method caused a reevaluation of the results in Taff (1983) and two extensions of it. Within the above

* By L. G. Taff

assumptions the analysis was extended to include quartic and quintic fits. In addition, the last assumption was relaxed for the cubic and quartic fit analysis so that the next leading terms (e.g., order 1/N terms) could be included. Doing so provides a more realistic guide when choosing the optimum time intervals, epochs, and orders of fit for real data. This Appendix summarizes these extensions of Taff (1983), mostly in graphical form.

A. Cubic Model

See Taff (1983) for the analytical preliminaries and presentation format. The estimate of the variance of the observed quantity x, say var[x(t)], has a local minimum at $\tau = 0$, local maxima at $\tau = \pm\tau_-$, and local minima at $\tau = \pm\tau_+$. The values of these quantities (to order 1/N) are

$$\tau_-^2 = \frac{N^2T^2}{5}(1 + 1/N), \quad \tau_+^2 = \frac{3N^2T^2}{7}(1 + 1/N)$$

The 1/N terms are illustrated in Fig. 1. This shows the normalized expected variance of x(t) as a function of $|\tau|/NT$ (it's an even function of τ) for $1/N = \infty$, 1/10, and 1/5.

The situation for var[$\hat{\dot{x}}(t)$] is simpler. It has a local maximum at $\tau = 0$ and local minima at $\pm\dot{\tau}$,

$$\dot{\tau} = \frac{NT}{\sqrt{7}}(1 + 1/2N)$$

The 1/N variation is illustrated in Fig. 2 in the same format as in Fig. 1. Figure 3 shows both var[$\hat{x}(t)$] and var[$\hat{\dot{x}}(t)$] ($N = \infty$) on the same page. Note that the expected variance of x(t) is fairly flat out to $\sim$ 0.7NT, rising steeply thereafter. The fluctuations in the expected variance of $\dot{x}(t)$ are larger and its takeoff is much more rapid.

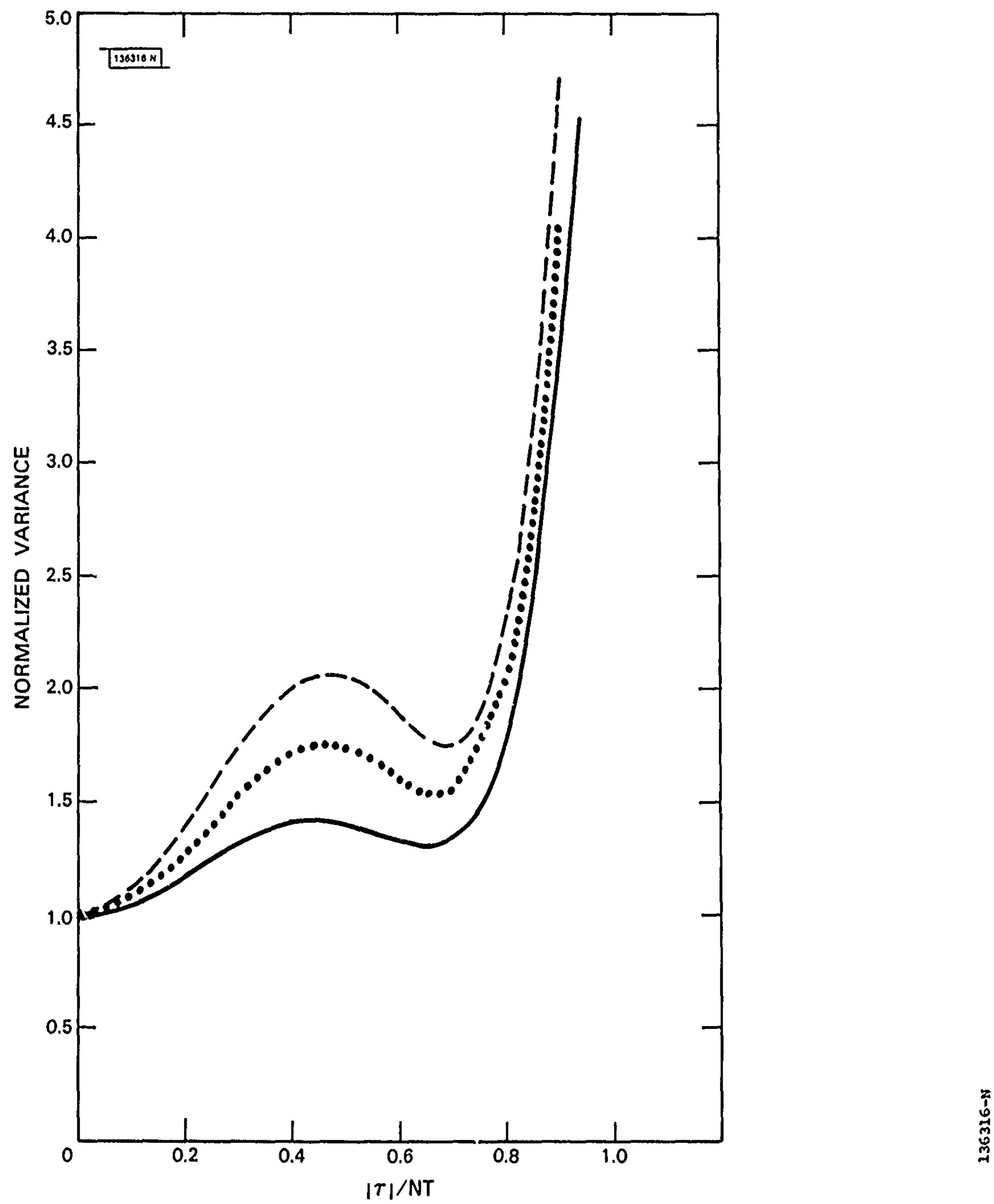

Fig. 1. Normalized variation of the expected value of x for a cubic polynomial fit. The full curve is the N = ∞ case, the dotted curve is the N = 10 case, and the dashed curve is the N = 5 case.

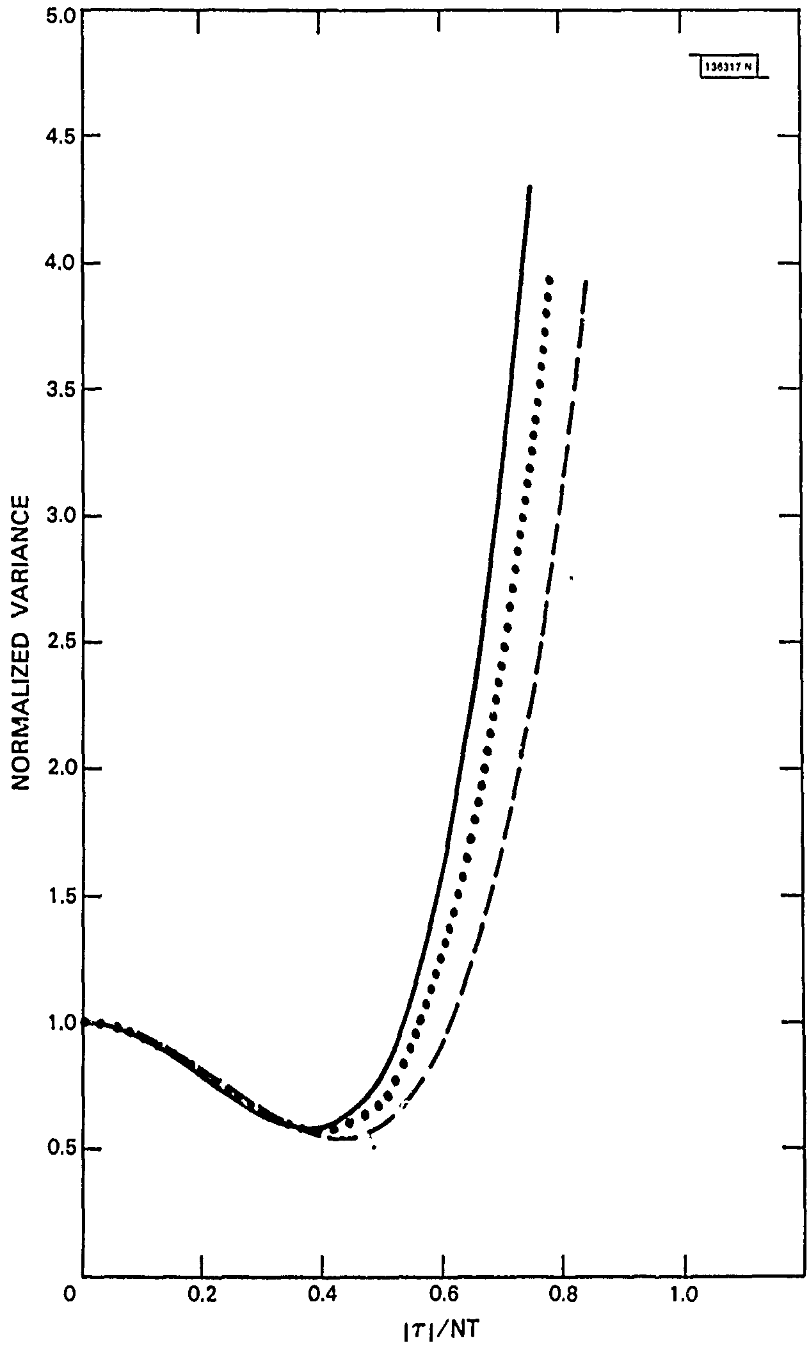

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Fig. 2. As Fig. 1 but for $\dot{x}$.

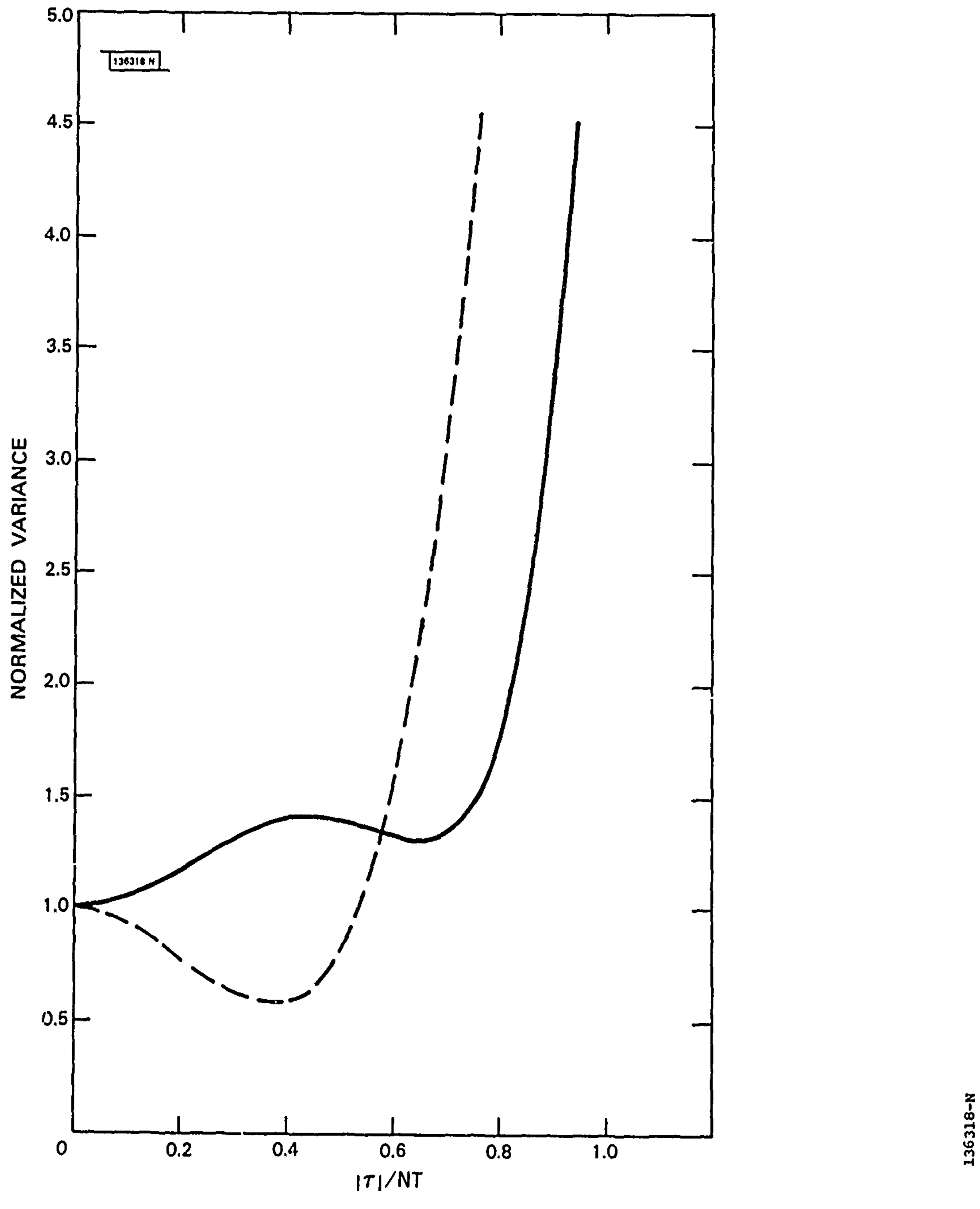

Fig. 3. The N = ∞ x (solid) and $\dot{x}$ (dashed) curves in Fig. 1.

B. Quartic Model

The model is

$$x(t) = a + b\tau + c\tau^2 + d\tau^3 + e\tau^4, \quad \tau = t - \langle t \rangle$$

The normal equations, MA = D, are, with the assumption of time symmetry,

$$M = \begin{pmatrix} S_0 & 0 & S_2 & 0 & S_4 \\ 0 & S_2 & 0 & S_4 & 0 \\ S_2 & 0 & S_4 & 0 & S_6 \\ 0 & S_4 & 0 & S_6 & 0 \\ S_4 & 0 & S_6 & 0 & S_8 \end{pmatrix}, \quad A = \begin{pmatrix} a \\ b \\ c \\ d \\ e \end{pmatrix}, \quad D = \begin{pmatrix} \Sigma\, x_n \\ \Sigma\, x_n \tau_n \\ \Sigma\, x_n \tau_n^2 \\ \Sigma\, x_n \tau_n^3 \\ \Sigma\, x_n \tau_n^4 \end{pmatrix}$$

Here $S_{2k} = w \sum_{n=-N}^{n=N} \tau_n^{2k} \to 2wN^{2k+1}\, T^{2k}/(2k+1)$ as $N\to\infty$. Define m and m' via

$$m = \begin{vmatrix} S_0 & S_2 & S_4 \\ S_2 & S_4 & S_6 \\ S_4 & S_6 & S_8 \end{vmatrix}, \qquad m' = \begin{vmatrix} S_2 & S_4 \\ S_4 & S_6 \end{vmatrix}$$

Then $|M| = mm'$ and

$$M^{-1} = |M|^{-1} \begin{pmatrix} (S_4S_8-S_6^2)m' & 0 & (S_4S_6-S_2S_8)m' & 0 & (S_2S_6-S_4^2)m' \\ 0 & S_6m & 0 & -S_4m & 0 \\ (S_4S_6-S_2S_8)m' & 0 & (S_0S_8-S_4^2)m' & 0 & (S_2S_4-S_0S_6)m' \\ 0 & -S_4m & 0 & S_2m & 0 \\ (S_2S_6-S_4^2)m' & 0 & (S_2S_4-S_0S_6)m' & 0 & (S_0S_4-S_2^2)m' \end{pmatrix}$$

One can compute that

$$|M|\ \mathrm{var}[\hat{x}(t)] = (S_4S_8-S_6^2)m' + [S_6m + 2(S_4S_6-S_2S_8)m']\tau^2 +$$

$$[(S_0S_8-3S_4^2+2S_2S_6)m' - 2S_4m]\tau^4 + [2(S_2S_4-S_0S_6)m' + S_2m]\tau^6$$

$$+ (S_0S_4-S_2^2)m'\tau^8$$

$$|M|\ \mathrm{var}[\hat{\dot{x}}(t)] = S_6m + 2[2(S_0S_8-S_4^2)m' - 3S_4m]\tau^2 + [9S_2m +$$

$$16\ (S_2S_4-S_0S_6)m']\tau^4 + 16\ (S_0S_4-S_2^2)m'\tau^6$$

Keeping the leading terms one discovers that

$$\mathrm{var}[\hat{x}(t)] \to \frac{25(1-1/N)}{2^7wN}\ [9-36(1-1/N)(\tau/NT)^2 + 294(1-2/N)(\tau/NT)^4$$

$$- 644(1-3/N)(\tau/NT)^6 + 441(1-4/N)(\tau/NT)^8]$$

$$\mathrm{var}[\hat{\dot{x}}(t)] \to \frac{25(1-3/2N)}{4wT^2N^3}\ [1+21(1-1/N)(\tau/NT)^2 - 105(1-2/N)(\tau/NT)^4$$

$$+ 147(1-3/N)(\tau/NT)^6]$$

It is straightforward, but laborious, to explicitly demonstrate that $\partial\mathrm{var}[\hat{x}(t)]/\partial\tau=0$ has three real roots. One finds, explicitly, that $\mathrm{var}[\hat{\dot{x}}(t)]$ has a relative minimum at $\tau=0$, relative maxima at $\pm$ NT(1+1/2N)/$\sqrt{7}$, and relative minima at $\pm$ NT(1+1/2N)/$\sqrt{3}$. Figure 4 illustrates the 1/N dependence of $\mathrm{var}[\hat{x}(t)]$, Fig. 5 that of $\mathrm{var}[\hat{\dot{x}}(t)]$, and Fig. 6 the N=$\infty$ versions of both. (As in Figs. 1 and 2, the curves are drawn for 1/N = ∞, 1/10, 1/5.)

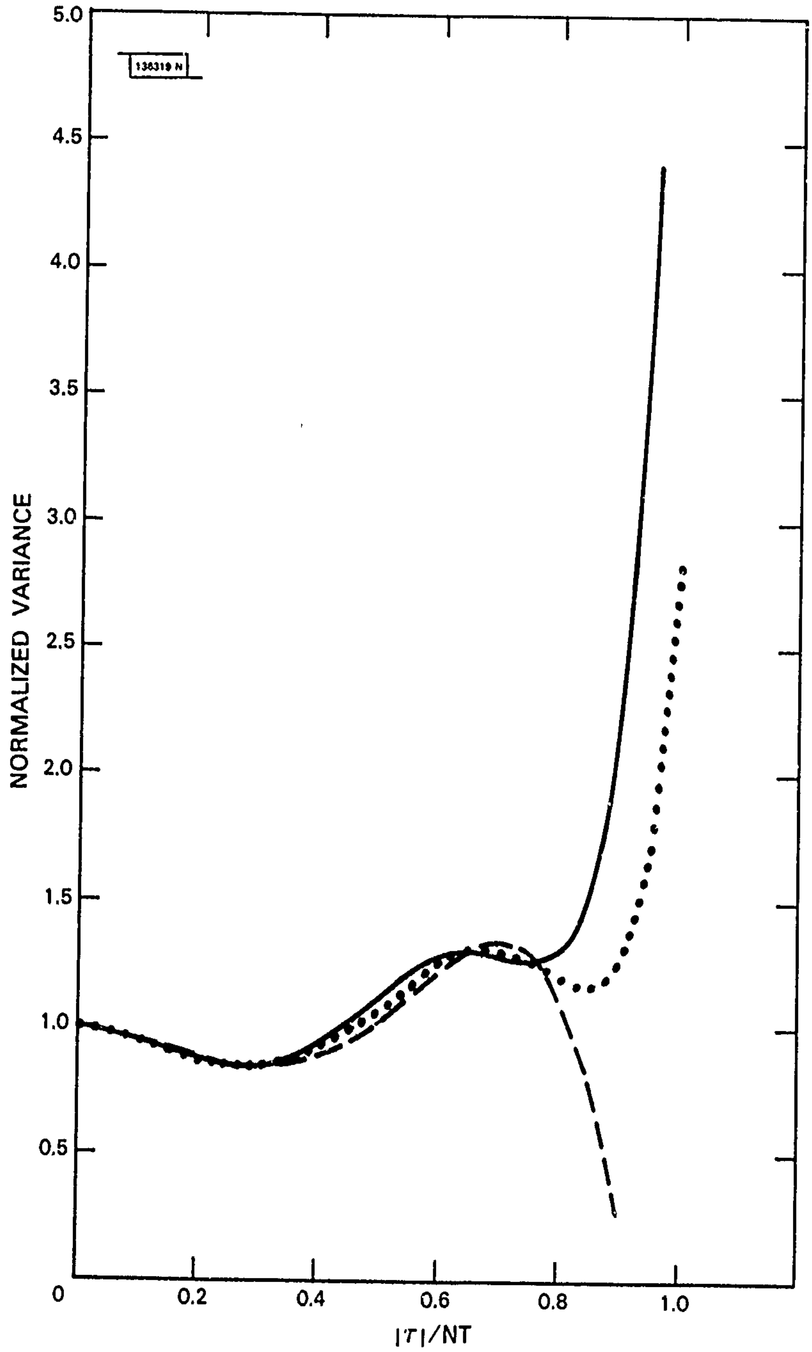

Fig. 4. As in Fig. 1 but for a quartic fit.

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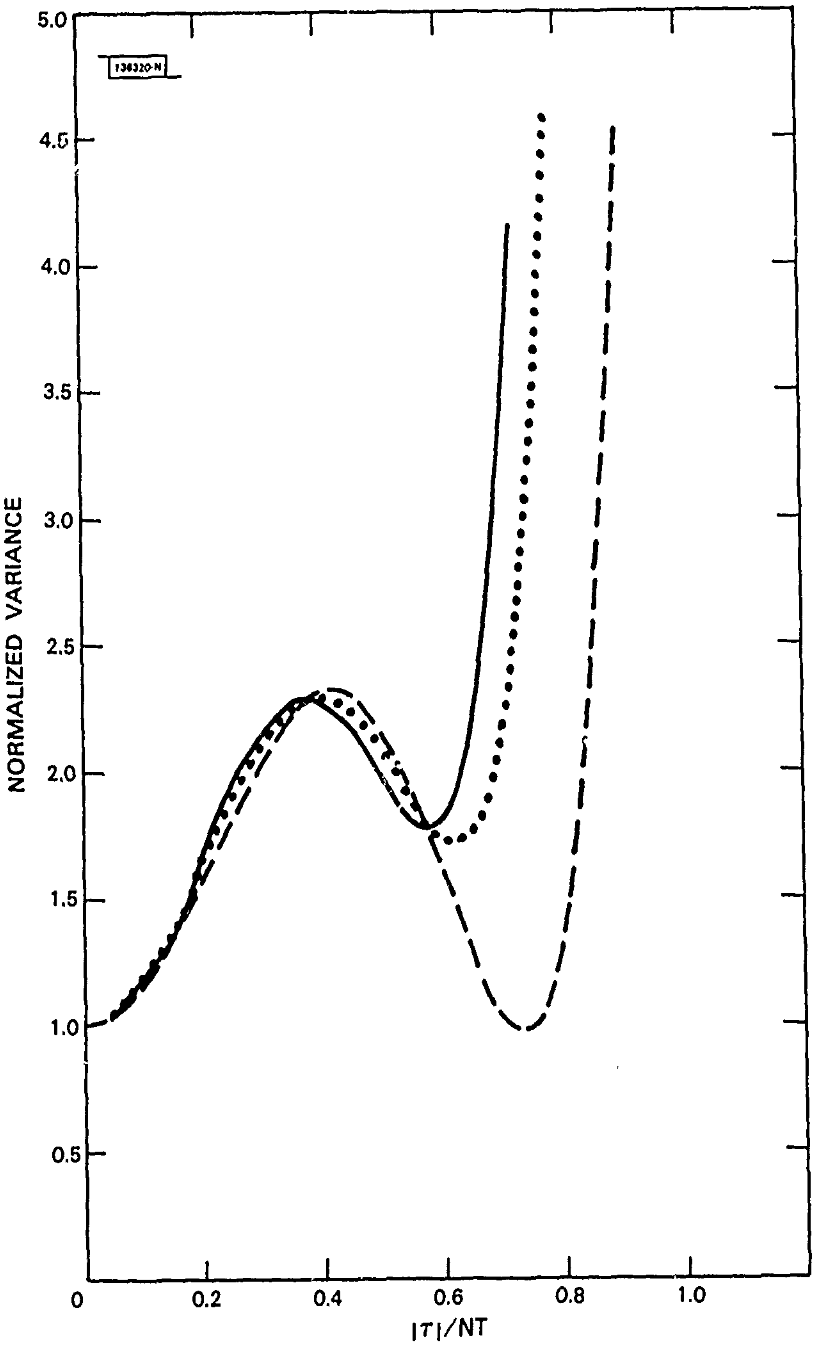

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Fig. 5. As in Fig. 2 but for a quartic fit.

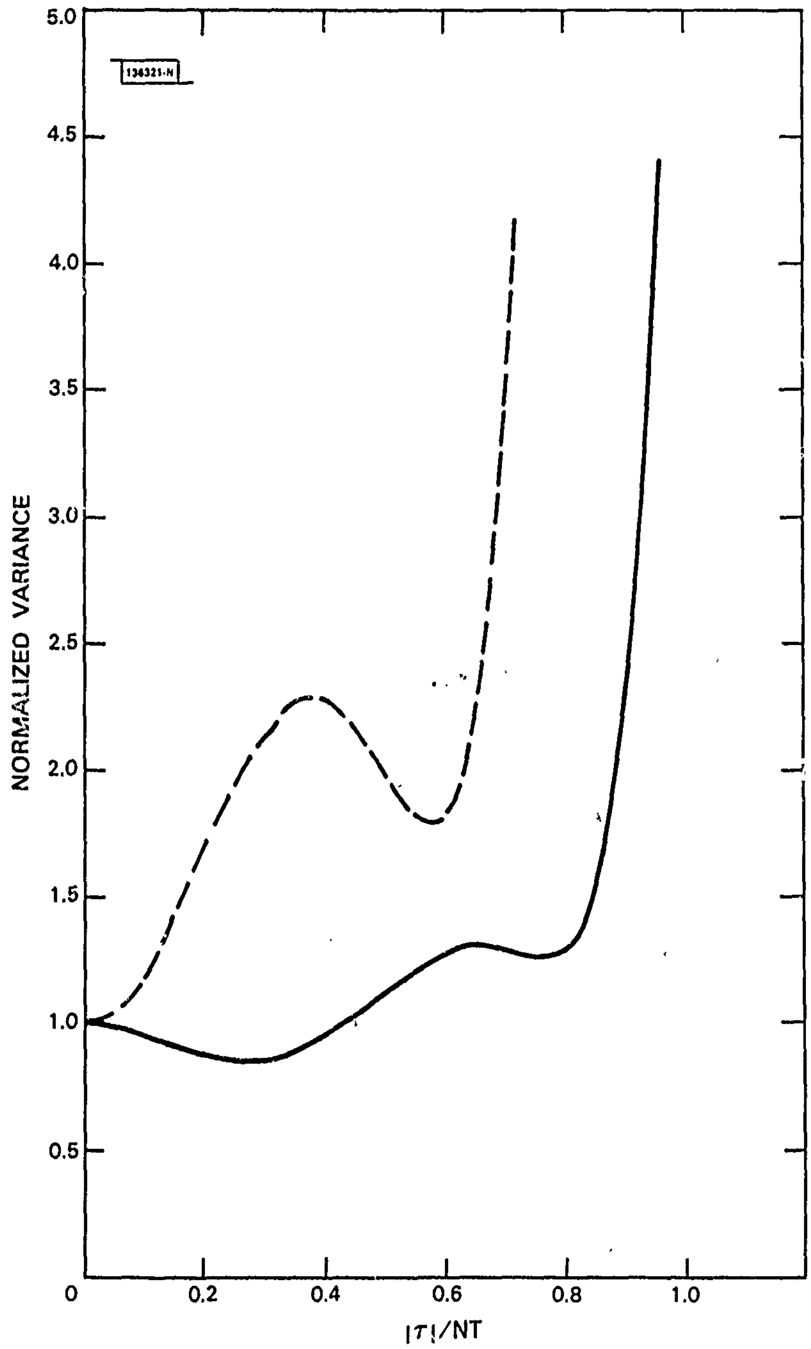

Fig. 6. As in Fig. 3 but for a quartic fit.

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The general characteristics of the curves for the quartic polynomial fit are similar to those of the cubic polynomial fit. The expected variance of x(t) curves are relatively flat out to $\sim 0.8T$ and then rise rapidly; those for the expected variance of dx/dt oscillate much more frequently and with greater amplitude before undergoing an extremely abrupt rise.

C. Quintic Model

The model is

$$x(t) = a + b\tau + c\tau^2 + d\tau^3 + e\tau^4 + f\tau^5, \quad \tau = t - \langle t \rangle$$

The normal equations take the form MA = D where

$$M = \begin{pmatrix} S_0 & 0 & S_2 & 0 & S_4 & 0 \\ 0 & S_2 & 0 & S_4 & 0 & S_6 \\ S_2 & 0 & S_4 & 0 & S_6 & 0 \\ 0 & S_4 & 0 & S_6 & 0 & S_8 \\ S_4 & 0 & S_6 & 0 & S_8 & 0 \\ 0 & S_6 & 0 & S_8 & 0 & S_{10} \end{pmatrix}, \quad A = \begin{pmatrix} a \\ b \\ c \\ d \\ e \\ f \end{pmatrix}, \quad D = \omega \begin{pmatrix} \Sigma x_n \\ \Sigma x_n\tau_n \\ \Sigma x_n\tau_n^2 \\ \Sigma x_n\tau_n^3 \\ \Sigma x_n\tau_n^4 \\ \Sigma x_n\tau_n^5 \end{pmatrix}$$

Keep m as before, but define m" as an augmented m',

$$m'' = \begin{vmatrix} S_2 & S_4 & S_6 \\ S_4 & S_6 & S_8 \\ S_6 & S_8 & S_{10} \end{vmatrix}$$

Then $|M| = mm''$ and the inverse of M is given by

$$|M|M^{-1} = \begin{pmatrix} (S_4S_8-S_6^2)m'' & 0 & (S_4S_6-S_2S_8)m'' & 0 & (S_2S_6-S_4^2)m'' & 0 \\ 0 & (S_6S_{10}-S_8^2)m & 0 & (S_6S_8-S_4S_{10})m & 0 & (S_4S_8-S_6^2)m \\ (S_4S_6-S_2S_8)m'' & 0 & (S_0S_8-S_4^2)m'' & 0 & (S_2S_4-S_0S_6)m'' & 0 \\ 0 & (S_5S_8-S_4S_{10})m & 0 & (S_2S_{10}-S_6^2)m & 0 & (S_4S_6-S_2S_8)m \\ (S_2S_6-S_4^2)m'' & 0 & (S_2S_4-S_0S_6)m'' & 0 & (S_0S_4-S_2^2)m'' & 0 \\ 0 & (S_4S_8-S_6^2)m & 0 & (S_4S_6-S_2S_8)m & 0 & (S_2S_6-S_4^2)m \end{pmatrix}$$

From these formulas and the usual expressions the $N \to \infty$ results for the expected variances are

$$\mathrm{var}[\hat{x}(t)] \to \frac{3^2 \cdot 5^2 \cdot 2^{-7}}{wN} [1 + 7\,(\tau/NT)^2 - 70\,(\tau/NT)^4 + 260.4\,(\tau/NT)^6$$

$$- 382.2\,(\tau/NT)^8 + 194.04\,(\tau/NT)^{10}]$$

$$\mathrm{var}[\hat{\dot{x}}(t)] \to \frac{3 \cdot 5^2 \cdot 7^2 \cdot 2^{-7}}{wT^2N^3} [1 - 12\,(\tau/NT)^2 + 126\,(\tau/NT)^4 - 348\,(\tau/NT)^6$$

$$+ 297\,(\tau/NT)^8]$$

The $N = \infty$ curves for each of these are shown in Fig. 7. Their behavior presents no new features.

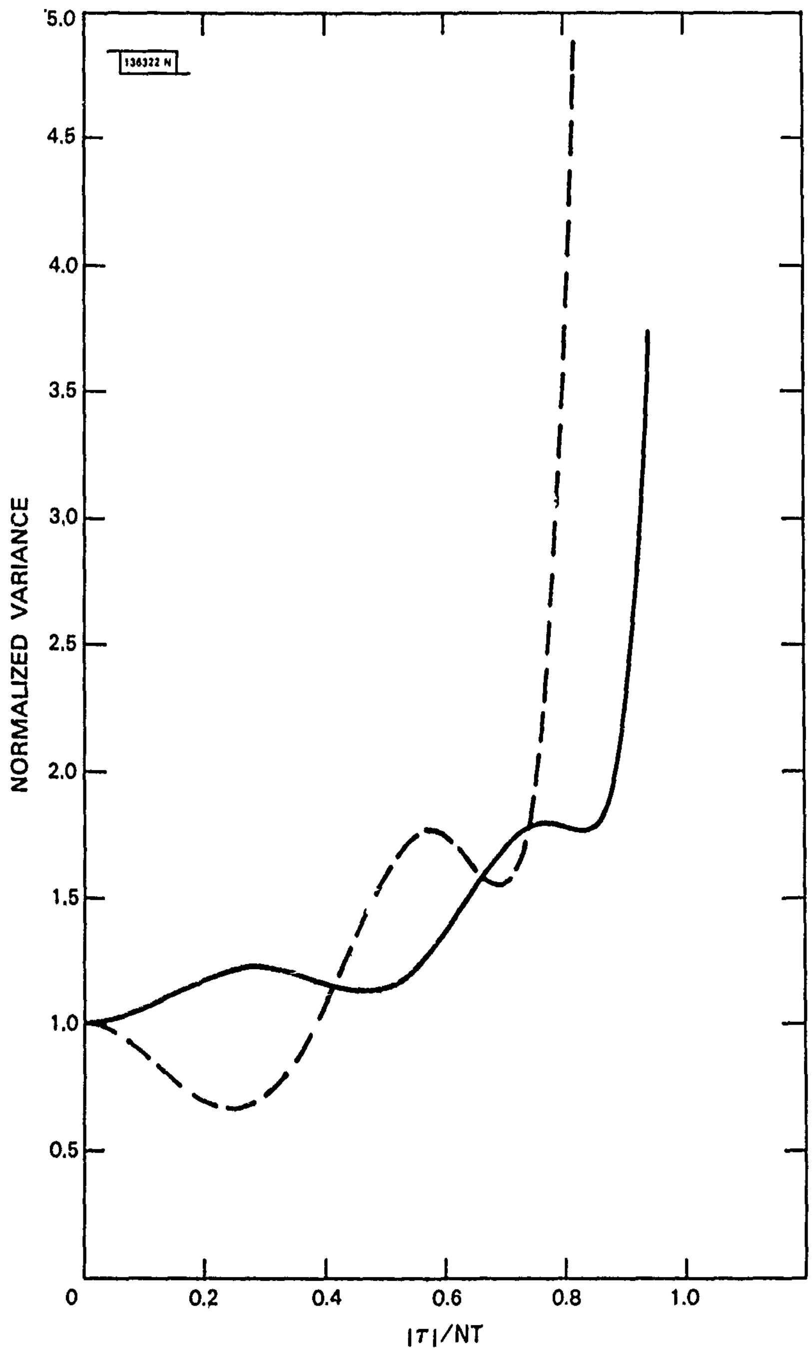

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Fig. 7. As in Fig. 3 but for a quintic fit.

REFERENCES

1. Brouwer, D. and Clemence, G. M., Methods of Celestial Mechanics (Academic, New York, 1961).
2. Gauss, F. G., Theoria Motus (Dover, New York, 1809).
3. Moulton, F. R., Astron. J. 23, 93 (1903).
4. Taff, L. G., "Optical Artificial Satellite Searches," Project Report ETS-44, Lincoln Laboratory, M.I.T. (1979a), DTIC AD-A071248/9.
5. Taff, L. G., "On Gauss's Method of Orbit Determination," Technical Note 1979-49, Lincoln Laboratory, M.I.T. (1979b), DTIC AD-A073776/7.
6. Taff, L. G., Publ. Astron. Soc. Pac. 93, 658 (1981).
7. Taff, L. G., "The Resurrection of Laplace's Method of Initial Orbit Determination," Technical Report 628, Lincoln Laboratory, M.I.T. (1983), DTIC AD-A126174/2.
8. Taff, L. G. and D. S. Hall, Cel. Mech. 16, 481 (1977).
9. Taff, L. G. and D. S. Hall, Cel. Mech. 21, 481 (1980).
10. Taff, L. G. and J. M. Sorvari, "Differential Orbit Correction for Near-Stationary Artificial Satellites," Technical Note 1979-38, Lincoln Laboratory, M.I.T. (1979a), DTIC AD-A074231/2.
11. Taff, L. G. and J. M. Sorvari, "Summary of Satellite Data Search Spring 79," Project Report ETS-45, Lincoln Laboratory, M.I.T. (1979b), DTIC AD-C018320.
12. Taff, L. G. and J. M. Sorvari, Cel. Mech. 26, 423 (1982).
13. Taylor, W. J., "Operations Manual Initial Orbit Estimate Module," Project Report ETS-41, Lincoln Laboratory, M.I.T. (1978), DTIC AD-A065255/2.

Note Added In Proof (February, 1984)

While this Report was being typed we more fully investigated the Taff-Hall method, especially with respect to a lower angular speed range. The problem is what is has always been, this approach is extremely sensitive to angular velocity errors. Within the context of the numerical experiments described herein the only available method of further improving the deduced angular velocities is to alter the nature of the numerical smoothing. We did this by splitting the total time span of the data in half and then separately performing the polynomial least squares fits (as outlined in the text). Now we return to the Taff (1983) methods and t_1, t_2 are the central epochs of the different time spans. The improvement in the element sets was dramatic.

The matrix below shows what happened to the three groups of results (NR = no root, NEL = root but no element set, EL = element set) after partitioning the observation span and then separately fitting the observations.

	One fit:	NR	NEL	EL
Two fits:	NR	6	0	0
	NEL	0	0	0
	EL	8	10	6
Totals		14	10	6

The 14 original "no root" cases are now 6 no root and 8 successfully determined, physically meaningful element sets. The 10 original "root but no element set" instances are all successful now too. More than this, the element sets themselves are materially better than before. Almost all of the systematic correlations are gone and the pointing errors are in the Gauss-Gibbs range.

Therefore, this method can be recommended if the data rate is very high (say 1 per 30 seconds) and precise ($\sim$1").

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angles-only	Laplace-Taff
satellites	Taff-Hall
asteroids	angular velocity methods

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

Over the past few years, passive, ground-based, angles-only initial orbit determination has had a thorough analytical, numerical, experimental, and creative re-examination. This report presents the numerical culmination of this effort and contains specific recommendations for which of several techniques one should use on the different subsets of high altitude artificial satellites and minor planets.

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